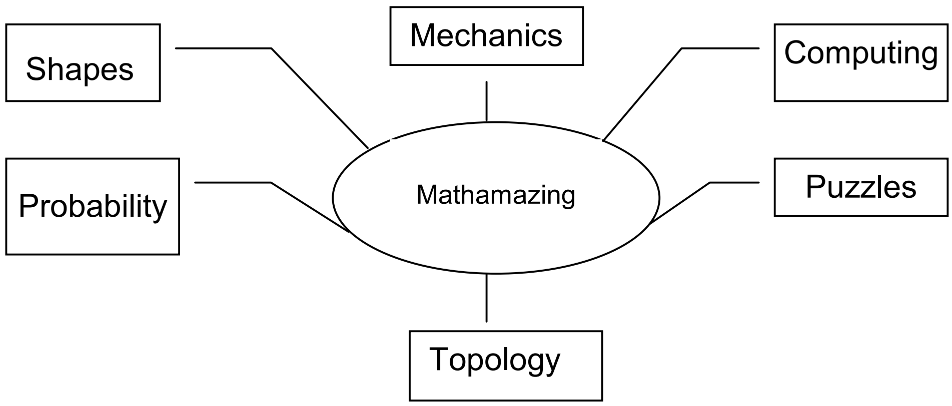
Mathamazing Exhibition
Secondary Students Maths Trail

Questacon

Teachers' Notes

The Questacon *Mathamazing* exhibition gives visitors the chance to explore and discover mathematics and its importance to everyday situations. Make a bridge, find your age in binary, roll a double six, dissect a Möbius strip or see how fast you can throw! The exhibition dispels the myth that 'maths is boring' by being lots of FUN! The hands on exhibits draw on a number of branches of mathematics:



Many of the exhibits draw connections to arithmetic or algebra. Of course, all are just starting points for further exploration at either home or school. Following are brief descriptions of the exhibits in the trail and some ideas for you to try with your class before and after your visit to *Mathamazing*. This should allow you to maximise the impact and value of the visit.

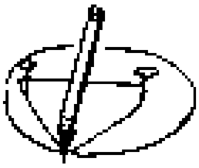
Featured in the exhibition are displays of famous and important mathematicians. Looking at the development of maths over the centuries shows the subject as a dynamic one, not something that was, is and always will be the same.

The *Mathamazing* Secondary Students' Maths Trail guides students through a range of exhibits, covering the topic areas outlined above. Seven exhibits from *Mathamazing* have been chosen for this trail. The trail challenges and develops students' skills in observation, prediction, estimation, logical reasoning and experimentation.

It is a good idea to split up your students into small groups of two or three. This facilitates communication about the exhibits and the ideas involved. Emphasise to your class the importance of each of them reading each exhibit's instruction panel, as well as his or her own trail sheet. Encourage students to come up with reasons for their answers. The group members should share the role of 'reporter', and all of the group should interact with all of the exhibits in the trail. While the exhibits on the trail are numbered, they can be visited in any order. You could start the groups at different exhibits to avoid crowding.

Before your visit you will need to photocopy enough worksheets for your students.

Ellipse (1)



This exhibit demonstrates the special relationship between the focal points of an ellipse. A ball is ‘launched’ down a ramp at one focus. The visitor can change the direction of the launch. The destination, however, is always the same. Every launch ends up with the ball falling into a hole placed at the other focus (as long as the table is level etc).

The equation of an ellipse is $x^2/a^2 + y^2/b^2 = 1$

The coordinates of one focus are $(-\sqrt{a^2 - b^2}, 0)$
while the coordinates of the other focus are $(+\sqrt{a^2 - b^2}, 0)$.

The ellipse is a conic section, first studied by the ancient Greeks. If you take a cone and slice it parallel to the base, you get a circle. If you cut it at an angle not quite parallel to the base you get an ellipse.

The exercise of the students’ sheet firstly aims to get them experimenting with the exhibit and observing accurately the path of the ball. It can be followed up at school by drawing an ellipse. This can be done by looping a piece of string around two pins stuck into a piece of card. Take a pencil and stretch the string taut, then move the pencil around both pins. Keep the string taut all the time to trace out an ellipse on the card. Perhaps the students could make their own elliptical “hole in one’ table, similar to the exhibit.

Naturally occurring ellipses include the paths following by planets on our solar system. Each planet has the sun at one focus of its elliptical path. Comets and artificial satellites also travel in elliptical paths.

And interesting anecdote: US President John Adams has an uncanny ability to second guess his opponents when sitting in Statuary Hall, Washington DC. The hall was elliptical in shape and Adams sat at one focus, his adversaries at another!

Find Your Age (2)

This exhibit shows a neat way of converting a decimal number (the visitor’s age) into its binary (base two) equivalent. The visitor checks a series of charts and tries to find his or her own age on some or all of them. Each chart corresponds to a different place value in binary code. Using a ‘1’ for ‘yes, my age appears in this chart’ and a ‘0’ for not appearing, the visitor can write out the binary form of his or her age.

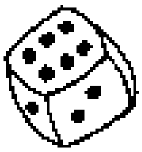
Ask students to create their own se of charts in binary. This will make it clearer to them how the charts were generated. They would start by completing a table like the one below:

Chart	A 2^6	B 2^5	C 2^4	D 2^3	E 2^2	F 2^1	G 2^0
1							1
2						1	0
3						1	0
4 etc					1	0	0

Binary numbers seen to be more complicated than decimal ones, but this is largely due to unfamiliarity with them. For computers the binary system is the ideal one to work with. Computers are made up of thousands of tiny switches. These switches can either be ‘off’ or ‘on’. These two states correspond to ‘zero’ and ‘’one’ in binary. Thus a string of these switches such as octal (base 8) and hexadecimal (base 16) are important in designing computers and electronic circuits.

Roll the Dice (3)

This exhibit deals with the very important areas of probability and chance. Visitors can examine the probabilities associated with rolling two six-sided dice.



Some probabilities are very obvious, such as the chance of getting a head in a toss of a coin (50%). What about tow heads in two tosses? (25%)

An interesting exercise in probability: How many people do you need in a room to have a greater than 50/50 chance that two of them will have the same birthday? The answer has been calculated as only 23. With 50 people it is almost certain that two will share a birthday. Check it with your class!

Decimal to Binary (4)

Like the ‘Find Your Age’ this exhibit utilises the binary (base two) number system. Students key in a number (up to 999) and the exhibit responds with its binary equivalent.

3-D Tic Tac Toe (5)

This exhibition is an extension of the traditional ‘noughts and crosses’ game. In addition to being three dimensional, the rows are four, not three, cells long.

For any game of tic tac toe the number of ways of winning is given by the formula:

No. of ways of winning $=1/2 \{(k + 2)^n - k^n \}$

Where k = no. cells on each side (in the exhibit this is 4)

N = no. of dimensions (in this case 3)

You can make a version of this at school. Perhaps 3 x 3 x 3 cells or even 5 x 5 x 5 x 5 instead of the 4 x 4 x 4 used in the exhibit. Possible materials include wood, plastic or metal. Make sure the holes are the right size for the balls you going to use, whether they are ping-pong balls, marbles or pebbles.

Manacles (6)

This exhibit is a demonstration of mathematical topology. It involves two visitors being put in rope manacles and these manacles being interlinked. The object is for the visitors to free themselves from each other, without taking the manacles off.

The task looks quite impossible and often ends up with the two participants on knots. The tangle can usually be undone quite easily by following these instructions:

Make a loop in your connecting rope and feed it between the strap and the wrist of your friend.

Now feed the loop over your friend’s hand.

Pull the double loop above the hand through the manacle. Pull your arms apart and you should be free! If you are still linked, try again!

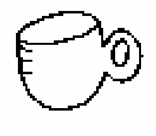
Topology is the study of shapes and surfaces. Topologists can relate somewhat different objects such as doughnuts and teacups, as they can (theoretically) be transformed into each other without breaking them apart. In the same way, a circle could be stretched into a square without breaking any lines. The manacles therefore, only looked as if they were knotted together.

Textile manufactures use topology to produce garments with special properties, such as knotted or crotched fabrics made from one continuous strand, or those that do not unravel where a fibre breaks.

Students could make some manacles with rope or string. They could try linking more than two people together and seeing if they can get themselves apart.

Cycloid (7)

This exhibit displays a special property of the cycloid. The cycloid is the fastest route between two points at different heights. The exhibit is a race between two marbles. One marble travels down an inclined plane, the other down a cycloidal track. The tracks are next to each other with the start and finish points on each track at the same height.



A ball rolling down a cycloidal path will move faster than one rolling down an inclined plane even though the inclined plane is a shorter distance.

Another property of a cycloid is illustrated in an exhibit which has a full cycloid. Cycloids are isochronous (from the Greek ‘same’ and ‘time’). Two balls released simultaneously from two different heights on either side of a cycloid will get to the bottom at the same time. Ask students to discuss why they think this happens.

The cycloid is the curve described by a point on the circumference of a rolling wheel. Your students can draw one by using an old plastic lid. They can make a hole as close as possible to the edge of the lid and stick a pencil lead through it. They then roll the lid along the edge of a ruler resting on a sheet of paper on a hard surface. The pencil will trace a cycloid on the paper. Alternatively they can attach a small light to the rim of a wheel, and then roll the wheel in a darkened room and watch the path of the light.